University of Central Florida
SPS Chapter Research 2017-18 Award Final Report

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Abstract

We, the Society of Physics Students at UCF, implement a high linearity silicon-based position sensitive detector along with piezo-electric polymer vibration sensors in a recreation of the Cavendish experiment. Measuring the gravitational constant along various millimeter range separation distances we attempt to verifying the position-dependence initially suggested by D. R. Long. By fitting our data to well known equations we are able extrapolate the values needed to solve for the gravitational constant. We found no indication that the gravitational constant holds any direct dependence towards the separation distance at the millimeter range. Our attempts at discriminating vibrational interference were unsuccessful, however we suggest methods for improving our idea.

Figure 1: The Society of Physics Students at UCF

1 Introduction

1.1 History

One of the more historically vibrant areas of research in physics is studying gravitational phenomenon. Dating back to the early 1600s and continuing to be an area of interest til this day. Galileo Galilei (1564-1642) was perhaps the first to begin experimental studies on the properties of gravity, however his contributions have become filled with stories that historians can not confirm to be true. One of such stories is that Galileo's interest in gravity was inspired by a swinging candelabrum during a Mass in the Cathedral of Pisa. He supposedly noted that the oscillation periods of the candelabrum remained relatively constant despite the decreases in the amplitudes. Galileo then proceeded to investigate this observation using stones of different weights and strings of different lengths, where he discovered that the periods were independent of the weight of the stones but dependent on the length of the strings. This finding directly contradicted the accepted belief at that time that heavier objects fell faster than lighter objects. This experimental finding lead to another of Galileo's famous but unverified stories, the dropping of objects from the Leaning Tower of Pisa. As the story goes; in an effort to prove his discovery to the students of the Aristotelian School, Galileo climbed to the top of the Leaning Tower of Pisa and the students watched as he dropped a heavy and light object. To the astonishment of the watching students the two object hit the ground at the same time. From there Galileo went on to develop a series of mathematical formulations for the motion of bodies.

The next significant contribution to the studies of gravity is highly contested, the prediction of the inverse square law. The debates between Robert Hooke (1635-

1703) and Isaac Newton (1642-1727) in both optics and gravitation are generally well-known, the continuation of this discussion will focus only on their debates in gravitation while attempting to properly credit significant discoveries. The complete derivation of the gravitational force is incontestably credited to Sir Isaac Newton, however credit must be given to those that aided in the philosophical discussion that lead to this discovery. From the preserved correspondences of Newton [19] we can see this discussion take place as follows. In 1679 Hooke wrote to Newton with the intent of initiating a philosophical correspondence on the topic of the celestial motions. This sparked a series of correspondences between the two, where the topic primarily focused on the motion of bodies governed by the force of gravity. In 1680 Hooke wrote to Newton "But my supposition is that the Attraction always is in a duplicate proportion to the Distance from the Center Reciprocall..." in response to a previous calculation Newton had sent him. After the submission of Newton's *Principia* in 1685 to the Royal Society, of which Hooke was a member, Hooke contested that Newton had not properly credited him. In 1686 Edmond Halley (1656-1742) wrote to Newton saying:

> ... Mr Hook has some pretensions upon the invention of ye rule of the decrease of Gravity being reciprocally as the squares of the distances from the Center. He sais you had the notion from him, though he owns the Demonstration of the Curves generated therby to be wholly your own ...

Newton addressed this concern stating that prior to receiving Hooke's correspondence in 1680 he had discussed the idea of the gravitational force being inversely proportional to the square of the separation distance with Christopher Wren (1632-1723). Newton also mentions that both Ismaël Boulliau (1605-1694) and Giovanni Alfonso Borelli (1608-1679) had suggested, without demonstrating, this similar idea. This debate resulted in the published version of Newton's *Principia* crediting Hooke, among others, for contributions to the derivation of the Inverse Square Law (ISL). Newton's calculations dictated the need for a constant to be introduced which he estimated the order of, however he never fully solved for it.

Much like Newton's estimation, preliminary calculations of the gravitational constant were based on measuring the Earth's mean density, since the constant is easily solved with this value as is shown in equation 1; where $R_\oplus$ is the Earth's radius, $\rho_\oplus$ is the Earth's mean density, $M_\oplus$ is the Earth's mass and the g is the gravitational acceleration on Earth. However, these calculations are typically downplayed due to their reliance on estimated values and subsequently experiments resulting in values for the gravitational constant were typically in pursuit of the mass of the Earth (or Earth's mean density).

$$G = g\frac{R_\oplus^2}{M_\oplus} = \frac{3g}{4\pi R_\oplus \rho_\oplus} \tag{1}$$

In 1778 Charles Hutton (1737-1823) reported the findings from the Schehallien experiment [15], an experiment that was initially proposed by Newton but dismissed due to his pessimistic view on it. The experiment was to have a pendulum placed near a large mountain, then measuring its deflection from its rest position. Hutton found that the Earth's mean density was $4\frac{1}{2}$ times the density of water, resulting in a value of $8.166 \times 10^{-11} \frac{m^3}{kgs^2}$ for the gravitational constant.

Henry Cavendish (1731-1810) was the first to yield accurate values for the gravitational constant, with his famous experimental study commonly referred to as 'the Cavendish Experiment' [4]. Since this is the very same experiment we have recreated, we only briefly discuss it here and later sections will cover it more extensively. The goal of his experiment was to measure the mass of the Earth, which would subsequently allow for the calculation of the masses of other celestial bodies. This experiment was first performed in 1798 using a torsion bar balance designed by John Michell (1724-1793), which had been specifically designed for measuring the mass of the Earth. However, Michell died before he was able to use it and so it was passed on to Cavendish, one of Michells friends. By suspending Michell's torsion balance then using two larger masses to cause a torque on it and measuring the angle of deflection, Cavendish was able to measure the gravitational constant. His findings resulted in a mean density of $\rho_\oplus = 5.448 \frac{g}{cm^3}$, and a gravitational constant of $G = 6.74 \times 10^{-11} \frac{m^3}{kgs^2}$. A remarkably accurate result, which differs by roughly 1% of the currently accepted value.

Since then numerous scientists have repeated both the Schehallien and Cavendish experiments, however it wasn't until 1891 that a value more accurate than Cavendish's was able to be measured. It was John Henry Poynting (1852-1914) who recreated the Cavendish experiment and measured a mean density of $\rho_\oplus = 5.49 \frac{g}{cm^3}$, and a gravitational constant of $G = 6.68 \times 10^{-11} \frac{m^3}{kgs^2}$. With the Cavendish experiments larger success rate in producing accurate measurements it has become the standard, through improvements on this experiment the accuracy of measurement has improved over the years. The current National Institute of Standards and Technology (NIST) recommended value is $G = 6.67408(31) \times 10^{-11} \frac{m^3}{kgs^2}$ [21].

Current studies of gravitational properties relay primarily on identifying the unification of relativistic physics with quantum mechanics. Experimentally this is done by conducting experimental tests of the ISL, since theorists have suggested that the gravitational force could behave different in short range separation distances [2,3,8]. These speculations come from the notion of more than 3 spatial dimensions existing, which are predictions made within string and M theory. Another theoretical suggestion of deviations in the ISL is the possibility of a Yukawa interaction between scalar particles such as dilatons and moduli [8–10,16]. There have also been several experimental and theoretical suggestions of deviations in the gravitational constant based on certain variables [20,22].

1.2 Project Relevance

Our project is a recreation of the original Cavendish experiment, however we utilize a high linearity silicon-based Position Sensitive Detector (PSD) and piezoelectric polymer vibration sensors to optimize the results of the experiment. Our experiment also makes use of a micro-controller and an attachment for it that allows for data collection to be done on the micro-controller. The relevance of our experiment with current research topics is the prevalence of torsion bar setups used in modern day gravitational experiments [1, 6, 12, 14, 17, 18, 23]. This implies that our project will directly contribute to the ongoing efforts to improve upon this experimental setup. We realize our project will not be able to produce results comparable to larger experimental groups with more funding, however our project will serve as benchmark of the new equipment and techniques we are employing. Providing researchers in larger groups with information on techniques that could work in their future experiments. As was shown in the history section contributing ideas and techniques can still have significant impacts even without significant results.

1.3 Intent

Our motives for completing this project are seen through three main goals; improving our research abilities as well as our presenting abilities, inspiring future generations of scientists as well as our peers, and contributing to the centuries-old study of gravity. In this section we will clearly outline our goals and briefly explain what work we have done to accomplish these goals, while later sections will cover our work more extensively.

The first of our goals was to improve our abilities, in both research and presenting. In physics its important to be able to identify a research question as well as devise a viable method of analyzing the question. Our hope with this project was to be able to gain some insight into working through the various phases of an experiment; design, building, testing and analysis. This became a goal of ours since although many of us work alongside research professors, we often don't get to be a part of all phases of an experiment. With respect to presenting, we aimed at giving talks on our experiences working on the project or on skills gained while working on this project. Throughout the year while we worked on this project we gained experience in using 3D drawing software, construction and assembly of equipment using tools that were new to some, using electronics such as amplifiers and micro-processors, and programming using Python. Multiple members of our group were also given the opportunity to give hour-long conference talks on topics they were familiar with but improved during our project. Said talks were given at an event inspired by our project and organized by our SPS chapter.

Another goal of ours was to inspire two distinct demographics, our peers and younger generations that might one day pursue careers in physics. The aim for our peers was to help bring to light the opportunities that are available for them, while also teaching them skills that could help them pursue these opportunities. This was accomplished with the event we inspired, which was previously mentioned, and multiple members of our group aided in organizing. Our objective for inspiring future scientists was to arrange for our group members to give talks at local high schools about their experiences working on this project. This was an important goal for us since we have come to realize students shy away from majoring in physics because they don't see how exciting it can be. We have unfortunately not had the opportunity yet to present at any high school, however we have begun communications with faculty members and hope to soon present our project.

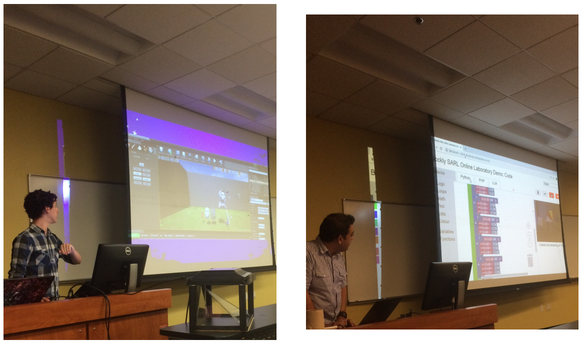
Our final goal was to contribute to the ongoing study of the gravitational constant, we knew our experiment could not provide for more accurate measurements but hoped it could give rise to new methods of conducting the classic Cavendish experiment. As is seen with the historical summary of the study of gravity, it is not only by producing results that one can contribute but also by providing others with new perspectives on old ideas. We accomplish this by devising a setup that, although builds off of previous designs, implements new features such as an attempt at discriminating vibration interference.

2 Outreach

Being that one of our motives with this project was to use it as an opportunity to teach and inspire others, in this section we outline the many ways our project has allowed us to perform outreach.

This project has aided us in convincing our university physics department to install SolidWorks in the undergraduate physics students study room, which has given students the chance to begin learning how to use this 3D computer-aided design (CAD) software. Experimental physics students often benefit from having experience using 3D CAD software, unfortunately few opportunities to practice using these software for free exist. This has given them a chance to familiarize themselves with SolidWorks.

Our project also inspired a conference-like event intended to teach others programming and circuit design skills that we noted as being very useful while working on our project. The event was the 1st ever UCF Raspberry Jam, organized by Brian C. Ferrari (Chapter research project leader) with the support of SPS chapter at UCF. It was a free event which managed to attract a diverse audience including undergraduate students, PhD candidates, and members of the professional community. It began with a series of workshops teaching Python coding and circuits, multiple talks given by members of the project, then ending with 30min presentations given on micro-processor related projects. Figure 2 shows two project presentations from the UCF Raspberry Jam; Brian Blalock, UCF student and member of Game Development Knights club, presenting a holographic video game which uses a micro-controller and Luis Felipe Zapata, PhD Candidate at FAU, presenting a micro-controller based remote laboratory program for online education courses.



(a) UCF student, Brian Blalock

(b) FAU PhD Candidate, Luis Felipe Zapata

Figure 2: Pictures of Project Presentations at UCF Raspberry Jam

We have already initiated communications with local high school teachers, our hope is to present our project to physics classes. Our presentation would cover the historical context of this experiment, mathematically formulations, construction of the experiment and the analysis of our data. The aim of our presentation will be to demonstrate how interdisciplinary physics research can be, since we've noted a large amount of physics students transfer into the major from engineering or computer science. We believe this to be because of the lack of awareness of physics interdisciplinary nature, so students will often pursue other majors. For example a student that enjoys programming and physics might pursue computer science because they enjoy coding more than lab work, without realizing they could pursue computational physics. We plan to give these talks before the end of the academic year.

3 Methods

3.1 Experimental Setup

Initially our setup was very simplistic, however we noted that wind flow from building A/C was preventing our light-weight torsion plate from reaching an equilibrium position. Out of the necessity to reduce the effects of air currents within the room a poly-carbonate chamber was built, along with new revised apparatus setup. Six 18×18 inch sheets of varied thicknesses were fused together into a cubic chamber using Methylene Chloride. This process first required machining the surfaces that would be bonded to provide maximum surface area, then clamping the sheets in the desired positions. Methylene Chloride was injected in-between bonding surfaces with a syringe, then the poly-carbonate sheets were left clamped until bonds solidified. A three-quarter inch thick sheet was used for the base, providing an increased amount of vibration damping, then two quarter inch and two eighth inch sheets were used for the sides of the chamber. The quarter inch thick pieces provided support for suspending necessary equipment (ie: PSD and cables). The top of the cubic chamber was an eighth inch thick UV-light damping sheet, the UV-light damping prevented damage to the PSD. Four two inch diameter vibration damping rubber soled feet were attached to the bottom of the cubic chamber, each at two inches away from each corner of the bottom sheet. A "track and cart" system was constructed and attached to opposite sides of the cubic chamber, allowing for the attractive masses to be moved into and out of testing position. The track was made from a 9 inch long piece of 80/20 T-slotted framing, while the cart was a 1 inch long piece. Cart was attached to the T-slot of the track, so that it could slide across, then it was attached to a handle on the outside of the chamber. Both sides of the chamber tangent to the track had 9 inch long slots cut into them using a vertical miller machine, allowing the handle to move them and lock them into place by tightening a hex screw. Figure 3 shows a partially incomplete model of our project, with most equipment and cables being ignored so that the setup is easier to understand.

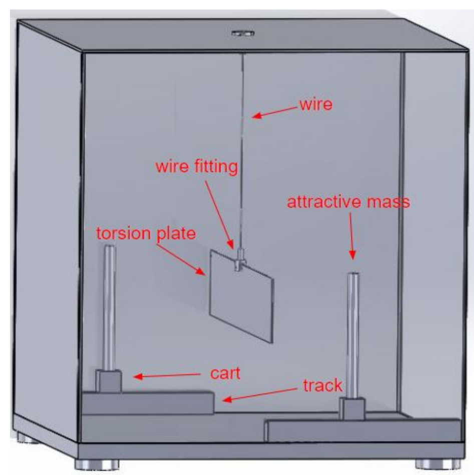


Figure 3: 3D CAD Model of Project

The top sheet of the chamber had four eighth inch holes drilled in the center of each side near the edge, then a half inch hole was drilled in the exact center of the top sheet to suspend the torsion plate from. The eighth inch holes were used to suspend the vibration sensor from, by threading the wires through the hole and connecting them into a breadboard atop the chamber. The sensors would then dangle from the roof of the chamber such that any vibration on the top sheet would cause them to vibrate and generate a signal. Near the base of the chamber, on the front face [1], 3 quarter inch holes were drilled into the sheet by the bottom left corner. The holes allowed for the vertical & horizontal adjustment knobs and power supply to connect to the laser module from outside the chamber. This allowed the freedom of being able to adjust the laser position without having to open the chamber up. The laser sat at the base of the chamber, while the PSD was suspended

[1]front denotes the face that holds both the PSD and laser

at the same height as the torsion plate; suspension was done by drilling a hole in the side of the chamber to screw the PSD module (or PSM) to wall of the chamber. Then a 1 inch hole saw was used to cut out a larger enough hole for the PSD's sub DB9 connection cable to pass through.

3.2 Procedure

This section serves to outline how we carried out our experiment, it will also serve as an instructions manual should our university decided to incorporate this experiment into one of the lab courses.

First ensure that all connections between equipment in the chamber and electronics outside of chamber are hooked up, then begin by powering on the following pieces of equipment in the order mentioned.

- Raspberry Pi 3 model B
- BitScope Micro Model 5
- PSM & Amplifier (connect PSM to amplifier then amplifier to power source)
- Launch BitScope application on Raspberry Pi
- Laser

Verify that both channels are active, channel 1 displaying PSD voltages and channel 2 displaying vibration sensor voltages. Using the horizontal and vertical adjustment knobs on the laser, align the optical path such that the voltage on channel 1 displayed by the BitScope is 0V. Once aligned, click the 'recorder' button located on the bottom right corner of the BitScope window. Using a hex key loosen the lock on each of the attractive masses, then maneuver them into their testing location. Tighten the lock and then allow for the experiment to continue, being sure to keep watch of the voltages displayed in channels 1 & 2. The experiment can be concluded once either the voltage displayed is constant or a sufficient number of oscillations have occurred for analysis to be performed. We recommend devising a method for extrapolating the equilibrium ϕ before conducting the experiment, so that you do not have to wait for the system to reach equilibrium (as that may take quite some time). Once sufficient data has been recorded, click the save button on the BitScope window and save the data into a external drive. The data will be saved as a CSV file, we then recommend using Python to analyze the data since Python has many modules (Pandas being our favorite) that make working with this file type easy. Then move the attractive masses to a neutral position, and allow for the torsion plate to return to its original position. Once the BitScope displays 0V in channel 1 again, the system is ready to begin another experiment. Repeat this process until a sufficient number of separation distances have been tested to identify if there exists a 'r dependence' in the gravitational constant. Finally power off equipment in reverse order of how they were powered on.

3.3 Calculations

The gravitational constant G will be calculated from the equation of motion of the torsion plate.

The angular form of Newton's second law is given by

$$\tau = \frac{d}{dt}L, \tag{2}$$

where τ is the net torque acting on the plate and L is its angular momentum. The two[2] torques acting on the plate are the torque due to the torsion cable, which obeys the angular form of Hooke's law and the torque due to the gravitational attraction force F_g for each of the tungsten rods. Since the moment arm of each of the two gravitational forces is half of the length of the plate, the net torque is

$$\begin{aligned} \tau = \frac{d}{dt}L &= -k\phi + F_g\frac{l}{2} + F_g\frac{l}{2} &(3)\\ &= -k\phi + F_g l, &(4)\end{aligned}$$

where k is the torsion coefficient of the cable and ϕ is the angle of rotation of the plate with respect to its neutral point[3]. Rewriting L as $I\frac{d}{dt}\phi$, where I is the moment of inertia of the plate (including the wire holder), and substituting in the full expression for the gravitational force[4], we obtain the differential equation

$$\frac{d^2}{dt^2}\phi + \frac{k}{I}\phi - G\frac{mM}{Ir^2}l = 0, \tag{5}$$

where m is the mass of one tungsten rod, M is the mass of the plate, and r is the distance between the plate and tungsten rod (in reality, r is a function of ϕ but we neglect this and assume it to be constant). This equation describes damped simple harmonic motion: the torsion plate will oscillate until an equilibrium is reached between the torque due to the torsion cable and that due to the gravitational attraction to the tungsten rods.

In principle, setting equation (4) equal to zero and using the measurement of ϕ when the plate is at equilibrium is sufficient to solve for G. However, given that the torsion coefficient k depends on the temperature and the amount of weight it holds among other variables, using it for calculations would introduce more uncertainties. Instead, we take advantage of the oscillatory motion that the plate undergoes before reaching equilibrium to compute G independently of k as follows.

The period of oscillation T, which is more easily measured than k, according to (5), is given by

$$T = 2\pi\sqrt{\frac{I}{k}}. \tag{6}$$

[2] Since the velocity of the plate is so low, the dominating component of the air resistance is that due to friction. Since the plate is thin, this air friction is negligible.

[3] Here, we make the distinction between neutral point and equilibrium point: neutral point refers to point at which the torsion cable has no angular tension in it while equilibrium point refers to the point at which the torque due to the cable matches the torque due to the gravitational pull of the tungsten rods, leaving the plate stationary.

[4] We are assuming spherical symmetry for simplicity. This is undeniably a source of error.

So, k can be expressed as

$$k = \frac{4\pi^2 I}{T^2}. \tag{7}$$

Taking equation (5) at equilibrium and substituting in the above expression for k, we have

$$G\frac{mM}{r^2}l = \frac{4\pi^2 I}{T^2}\phi, \tag{8}$$

which can be solved for G to find

$$G = 4\pi^2 r^2 \frac{I}{lmM}\frac{\phi}{T^2}. \tag{9}$$

The moment of inertia I is the sum of the moments of inertia of the torsion plate I_{plate} and the cable holder I_{holder},

$$I = I_{\text{plate}} + I_{\text{holder}}. \tag{10}$$

The moment of inertia of the plate is given by

$$I_{\text{plate}} = \frac{1}{12}M(l^2 + d^2), \tag{11}$$

where d is the thickness of the plate.

The wire holder, on the other hand, will be modeled as three concentric annular cylinders stacked on top of each other, with the bottom-most cylinder having half of its circumference removed. In general, the moment of inertia of an annular cylinder of mass m with inner radius r_1 and outer radius r_2 is $\frac{1}{2}m(r_1^2+r_2^2)$. So, we can express I_{holder} as

$$I_{\text{holder}} = \frac{1}{2}m_{\text{t}}(r_{\text{t1}}^2 + r_{\text{t2}}^2) + \frac{1}{2}m_{\text{m}}(r_{\text{m1}}^2 + r_{\text{m2}}^2) + \frac{1}{4}m_{\text{b}}(r_{\text{b1}}^2 + r_{\text{b2}}^2), \tag{12}$$

where the subscript t denotes quantities for the top annular cylinder, m for the middle, and b for the bottom. Note that the coefficient in the term corresponding to the bottom cylinder is 1/4 since its moment of inertia is half of a complete annular cylinder.

Thus, G can be written as

$$G = 4\pi^2 r^2 \frac{\frac{1}{12}M(l^2 + d^2) + I_{\text{holder}}}{lmM}\beta, \tag{13}$$

where the complete expression for I_{holder} is given by (12) and we introduce $\beta \equiv \frac{\phi}{T^2}$, the significance of which will be clarified in the data analysis.

3.4 Data Collection and Analysis

All data collection was done through a Raspberry Pi 3 model B with a BitScope Micro Model 5 attachment which was used to record data from equipment. The BitScope attachment has a 20 MHz Digital Oscilloscope feature with 12-bit analog sample resolution, samples to 40 MS/s, 12 kB buffer, 3.0 Mb/s. Torsion plate deflection data was collected by an On-Trak Photonics inc. PSM 1-10 Linear Silicon detector with a 10 × 2 mm sensor area and a 250nm resolution. Then having the signal generated by the PSD sent to an On-Trak Photonics inc. OT-301SL Single Axis PSD amplifier set to a gain of $10^{-3}\frac{A}{V}$, a 0V bias and no applied X-offset. Amplified PSD signal is specified to have a ±1% linearity and 16kHz frequency response. The amplified signal was then connected to the primary channel of the BitScope attachment. Vibration data was collected by connecting all sensors signal outputs in series on a breadboard. The culminated signal was then connected to the secondary channel of the BitScope attachment. During testing all voltages measured by the BitScope were recorded in a comma separated values (CSV) file and stored in a USB.

Collected data was analyzed through Python code, where fittings and extrapolations were performed. Vibration sensor data was reduced by using a sigma rejection routine that would zero out all recorded values that were within 3σ (standard deviations) of the mean data value. This served to isolate the 'real' signals from the background signals that the sensors typically generate. The remaining signals were then stored in an array, including both signal amplitude and time of event, for use in PSD data analysis. However, in most cases the vibration sensors did not generate any 'real' signals; only when forcefully causing vibrations to the chamber did we record 'real' signals. Had we been able to record 'real' signals our discrimination plan was to take the time of the event and use it as a reference point. We would then have done all fittings and extrapolations using data collected between start of the experiment and timing of the signal generated. Then this result would be cross-checked with the total data fittings and extrapolations to identify the severity of the vibrations interference with the experiment.

PSD data was fitted to two distinct equations: the exponentially decaying sinusoid equation and the exponential decay equation (of the previous equation); both can be seen in the following equation.

$$x(t) = A_o e^{-\gamma t}\cos(\omega t - \alpha) \tag{14}$$

Once our data was fit to these parameters we identify the equilibrium position value, ϕ, through a multi-point extrapolation. Oscillation periods were found using the exponentially decaying sinusoid fit as a reference, we take only the turning point values that correspond with peaks in the fitted curve. This was done because our PSD data was not smooth, it had slight increases and decreases that did not correspond to turning points in the fitted data. These fitted turning points were used to calculate the mean oscillation period. The corresponding β value was immediately calculated for in our program. It should be noted that our data analysis code revolved exclusively around finding the variables need to solve for β, this is because all other variables in the equation for the gravitational constant, when held constant, can be ignored when attempting to determine the separation distance dependence of the gravitational constant.

4 Results and Discussion

Figure 4 displays β values found for different distances with β-related variables calculated from direct measurements as mentioned in the previous section. Only β

and the separation distance are measured while all other variables are assume to be constant, theoretical calculations were made using the same constant variables as were used in the experimental calculations.

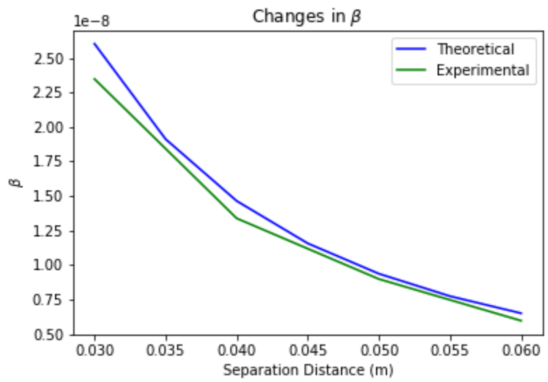


Figure 4: Plotted changes in β across various millimeter separation distances

From figure 4 we can see that our experimental values show the same trend as the theoretical values, found by calculating β from the NIST recommended gravitational constant value. Some minor discrepancies can be seen between both values however not significant enough to be indicative of a dependence on the separation distance. These discrepancies most likely arise from errors in our experiment, which we discuss solutions for in the conclusion section. It should be noted that more points are plotted along the theoretical curve, at $\{0.03, 0.035, 0.04, 0.045, 0.05, 0.055, 0.06\}$, than the experimental, at $\{0.03, 0.04, 0.05, 0.06\}$. The lines serve only as guides for the eye, since no fitting was applied they do not accurately demonstrate the nature of the curve, only points plotted along the given x-axis ranges are actual data points while the lines just linearly connect these points.

5 Conclusion

We found no indication that the gravitational constant holds any direct dependence towards the separation distance at the millimeter range. Our attempts at discriminating vibrational interference were unsuccessful, however after careful consideration and thoughtful deliberation we have determined steps that can be taken to improve upon our idea. Suspending a mass from each of the vibration sensors would increase the signal generated from the sensors during small vibrations, allowing a more thorough analysis to be done. Possible improvements on the experiment as a whole would be to include more points of during data collection, so that we can more accurately visualize the trends formed on the $\beta(r)$ curve.

6 Expenses

EQUIPMENT	AMOUNT	SUPPLIER	NOTES
Vibration sensors (5)	\$37.28	TTI	
Tungsten rods (2)	\$122.54	McMasterCarr	
Tungsten wires (1)	\$117.74	Spectrum Scientifics	
PSD (1)	\$88.00	On-Trak Photonics Inc.	1L10
PSM (1)	\$288.00	On-Trak Photonics Inc.	PSM1-10
Signal Amplifier (1)	\$695.00	On-Trak Photonics Inc.	OT-301SL
On-Trak Shipping	\$37.00	On-Trak Photonics Inc.	
Poly-Carbonate Sheets (6)	\$96.60	Acme Plastics	
BitScope Micro Model 5 (1)	\$150.00	BitScope Designs	BS05
Vibration Damping Feet (4)	\$10.99	Amazon	
Wire Fitting (2)	\$12.99	Amazon	
Stock Metal	\$65.86	McMasterCarr	
Total	\$1,722.00	Remaining Funds	\$278.00

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